complete differential equations? In the table below some of the experimental data is provided so that interested readers may draw their own conclusions.

Table 1.

x (mm)	Nu_x	Gr_x	ΔT (°C)
25·4	4·8	$2{\cdot}98 \times 10^{4}$	14·0
12·7	3·1	$3{\cdot}72 \times 10^{3}$	14·0
8·75	2·66	$1{\cdot}2 \times 10^{3}$	14·0
4·60	2·22	$1{\cdot}66 \times 10^{2}$	14·0
12·7	3·2	$5{\cdot}2 \times 10^{3}$	19·5
11·1	2·95	$2{\cdot}76 \times 10^{3}$	19·5
6·8	2·77	$8{\cdot}0 \times 10^{2}$	19·5
3·95	2·62	$1{\cdot}58 \times 10^{2}$	19·5
12·7	3·5	$6{\cdot}46 \times 10^{3}$	25·8
8·75	3·1	$2{\cdot}1 \times 10^{3}$	25·8
4·75	2·82	$3{\cdot}42 \times 10^{2}$	25·8

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INERT-GAS TRANSPORT IN LIQUID METALS DURING BOILING EXPERIMENTS

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NOMENCLATURE

C_A,	molar concentration of inert gas in liquid solution;
C_{A1},	molar concentration of inert gas in liquid solution at $t \geqq 0$ and $X = X_2$;
C'_A,	molar concentration of inert gas in surface cavities;
C_B,	molar concentration of liquid in liquid solution;
C'_B,	molar concentration of vapor in surface cavities;
D_{AB},	mass diffusivity coefficient of inert gas in liquid solution;
D'_{AB},	mass diffusivity coefficient of inert gas in vapor;
K,	Henry's Law constant;
M_B,	molecular weight of liquid;
P_A,	partial pressure of inert gas in surface cavity;
P_{A1}, P_{AG},	partial pressure of inert gas in gas blanket;
$P_A(t)$,	partial pressure of inert gas in surface cavities at time t;
P_{gb},	pressure at liquid–inert gas blanket interface;
T,	liquid temperature;
T_{gb},	temperature of liquid at inert-gas blanket–liquid interface;
T_0,	initial liquid temperature;
t,	time;
V_A,	molar volume of A in liquid at normal boiling point;
X,	coordinate in isothermal system;
X_A,	mole fraction of inert gas in liquid solution;
X_{gb},	mole fraction of inert gas in liquid sodium at blanket–liquid interface;
X_3,	liquid metal pool depth;

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X_1,	nominal surface cavity depth;
X_2,	length defined as $X_2 = X_3 + X_1$;
Y,	axial coordinate in flowing system.

Greek symbols

μ,	viscosity of the solution;
ξ,	dimensionless coordinate defined in equation (19);
ρ_B,	density of liquid;
τ,	dimensionless time defined in equation (20);
$\phi, \phi(\xi, \tau)$,	dimensionless concentration difference defined in equation (18);
ψ_B,	"association parameter" (equal to unity for an unassociated solvent).

INTRODUCTION

DURING incipient-boiling superheat tests with sodium, an inert-cover-gas blanket is generally employed in the system [1–4]. Fast reactor systems also include a free sodium surface exposed to an inert gas [5, 6]. A model which predicts the incipient-boiling superheat from knowledge of the pressure-temperature history of the boiling surface has been proposed [1, 7]. This model has shown agreement with experimental data when the partial pressure of inert gas in the surface cavities of the boiling surface is known [8]. However, in most experimental tests, the partial pressure of inert gas in the surface cavities is not known accurately both during the establishing of a pressure-temperature history and during the actual boiling tests [9]. Much question has existed over the fact that some of these experimental data [2, 10] do not agree with the prediction of the pressure-temperature history model when the inert-gas partial pressure in the surface cavities is assumed zero. The proposed reason for this disagreement is that inert gas has been transported into the surface cavities of the heater surface; hence, the partial pressure of the inert gas in the heater surface cavities is significantly greater than zero. In order to demonstrate the existence of an inert-gas partial pressure in the surface cavities, the following inert-gas diffusion model is presented and solved.

DEVELOPMENT OF MODEL

One of the difficulties when establishing a pressure-temperature history in a system by means of applying an inert-gas pressure in the blanket region and when attempting to conduct superheat tests in systems employing inert-cover gas is the transport of this inert gas to surface cavities. Transport of the inert gas from the blanket is accomplished by diffusion of the inert gas in the liquid solution and may also be aided by convection of the liquid providing sufficient temperature gradients exist in the system. In order to examine the diffusion of an inert gas into surface cavities, the mathematical description of a simplified, one-dimensional model is proposed. The solution of this model yields the inert-gas partial pressure in the surface cavities as a function of time. Figure 1 illustrates the physical model.

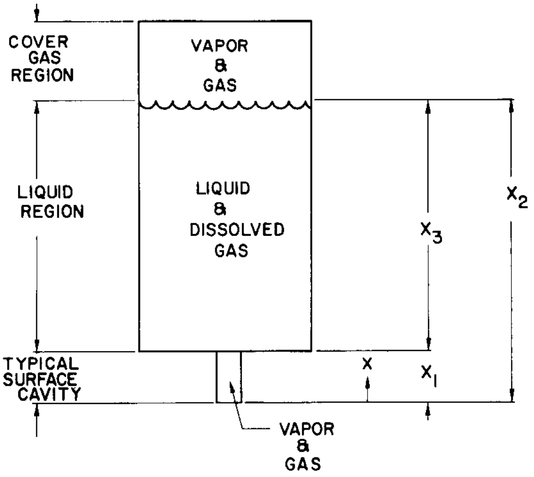

FIG. 1. Physical model for inert-gas diffusion.

Assuming that molecular diffusion is the only mode of mass transfer, the conservation equations for the three regions may be expressed:

Gas Blanket

$$\text{for } t < 0, \quad P_{AG} = 0 \tag{1}$$

$$\text{for } t \geqslant 0, \quad P_{AG} = P_{A1} \tag{2}$$

Liquid Solution

$$\text{for } t < 0, \quad C_A = 0 \tag{3}$$

$$\text{for } t \geqslant 0, \quad \frac{\partial C_A}{\partial t} = D_{AB}\frac{\partial^2 C_A}{\partial X^2} \tag{4}$$

Surface Cavity

$$\text{for } t < 0, \quad P_{AG} = 0 \tag{5}$$

$$\text{for } t \geqslant 0, \quad \frac{\partial C'_A}{\partial t} = D'_{AB}\frac{\partial}{\partial X}\left[\left(C'_A + C'_B\right)\frac{\partial}{\partial X}\left(\frac{C'_A}{C'_A + C'_B}\right)\right]. \tag{6}$$

At the liquid–vapor interface, the partial pressure of the inert gas is related to the mole-fraction of the dissolved gas in the liquid by Henry's Law,

$$X_A = K(T)\,P_A. \tag{7}$$

For $t \geqslant 0$, the concentration of the inert gas at the gas blanket–liquid interface is

$$\frac{C_A(X_2, t)}{C_A(X_2, t) + C_B(X_2, t)} = K(T)\,P_{A1}. \tag{8}$$

Since $C_B(X_2, t) \gg C_A(X_2, t)$, and

$$C_B = \frac{\rho_B}{M_B}. \tag{9}$$

equation (8) can be expressed,

$$C_A(X_2, t) = \frac{\rho_B P_{A1} K(T)}{M_B}. \tag{10}$$

Now, a similar relationship exists between the partial pressure of the inert gas in the cavity and the concentration of inert gas in the liquid solution at $X = X_1$. This relationship can be expressed,

$$C_A(X_1, t) = \frac{P_A \rho_B K(T)}{M_B}. \tag{11}$$

The boundary conditions which need be employed to obtain a solution to equations (2), (4) and (6) are

$$\text{(1) for all } t \text{ and } X,\ T = T_0 \tag{12}$$

$$\text{(2) at } t = 0,\ C_A(X, 0) = 0, \text{ and } P_A = 0 \tag{13}$$

$$\text{(3) at } X = X_2 \text{ and } t \geqslant 0,\ C_A(X_2, t) = \frac{\rho_B P_{A1} K(T_0)}{M_B} = C_{A1} \tag{14}$$

$$\text{(4) at } X = X_1 \text{ and } t \geqslant 0,\ D_{AB}\frac{\partial C_A}{\partial X} = D'_{AB}(C'_A + C'_B)\frac{\partial}{\partial X}\left(\frac{C'_A}{C'_A + C'_B}\right) \tag{15}$$

$$\text{(5) at } X = 0,\ \frac{\partial}{\partial X}\left(\frac{C'_A}{C'_A + C'_B}\right) = 0. \tag{16}$$

At this point, the mathematical description of the problem is totally specified.

METHOD OF SOLUTION

Typical surface cavity dimensions are orders of magnitude less than the other system dimensions illustrated in Fig. 1. Hence, the capacity for a cavity to accumulate inert gas can be considered negligible. Thus, the boundary condition at $X = X_1$ can be simplified to

$$\text{at } X = X_1,\ \frac{\partial C_A(X_1, t)}{\partial X} = 0. \tag{17}$$

In order to solve the equation describing the diffusion in the liquid solution, i.e. equation (4), the following dimensionless quantities are defined:

$$\phi(\xi, \tau) = \frac{C_A - C_{A1}}{C_{A1}}, \tag{18}$$

$$\xi = \frac{X_2 - X}{X_3}. \tag{19}$$

and

$$\tau = \frac{t D_{AB}}{X_3^2}. \tag{20}$$

Employing these dimensionless relationships, the equation and boundary conditions describing the transient mass transfer in a pool of liquid sodium following a step addition in cover gas is:

$$\frac{\partial \phi}{\partial \tau} = \frac{\partial^2 \phi}{\partial \xi^2} \tag{21}$$

$$\text{at } \tau = 0, \quad \phi = -1 \tag{22}$$

$$\text{at } \xi = 0, \quad \phi = 0 \tag{23}$$

$$\text{at } \xi = 1, \quad \frac{\partial \phi}{\partial \xi} = 0. \tag{24}$$

The entire system is now described by equations (21)–(24).

Using the method of solution outlined in [11], the following solution can be obtained:

$$\phi(\xi, \tau) = -\frac{2}{\pi}\sum_{m\,\text{odd}} e^{-(\pi m/2)^2 \tau}\left(\frac{1 - \cos \pi m}{m}\right)\sin\frac{m\pi\xi}{2}. \tag{25}$$

Equation (25) describes the diffusion of inert-cover gas in the liquid solution after a step-wise addition of inert-gas partial pressure in the gas blanket.

ANALYSIS OF INERT-GAS DIFFUSION

While changing the pressure of the inert-cover gas, it is important to know the transient partial pressure of the cover gas in the surface cavities in order to accurately determine the radius to which the liquid can penetrate into the cavity. The diffusion of the inert-cover gas in the liquid solution after a step-wise addition of inert-gas partial pressure in the gas blanket can be described by equation (25). In order to solve for the partial pressure in the surface cavity, as a function of time, equation (25) must be solved at $\xi = 1$ for all τ. Equation (25) can now be expressed,

$$\phi = -\frac{4}{\pi}\{e^{-2{\cdot}465\tau} - \tfrac{1}{3}e^{-22{\cdot}19\tau} + \tfrac{1}{5}e^{-61{\cdot}6\tau} - - - -\}. \tag{26}$$

It is seen that the higher order terms damp out very rapidly and that for reasonable values of τ (i.e. $\tau > 0{\cdot}1$) only the fundamental mode exists. Hence, ϕ can be approximated by the expression

$$\phi = -\frac{4}{\pi}e^{-2{\cdot}465\tau}. \tag{27}$$

The diffusion coefficient, D_{AB}, can be calculated by the expression proposed by Wilke and Chang [12] which is valid for small concentrations of A in solution B:

$$D_{AB} = 7{\cdot}4 \times 10^{-8}\frac{(\Psi_B M_B)^{0{\cdot}5} T}{\mu V_A^{0{\cdot}6}}. \tag{28}$$

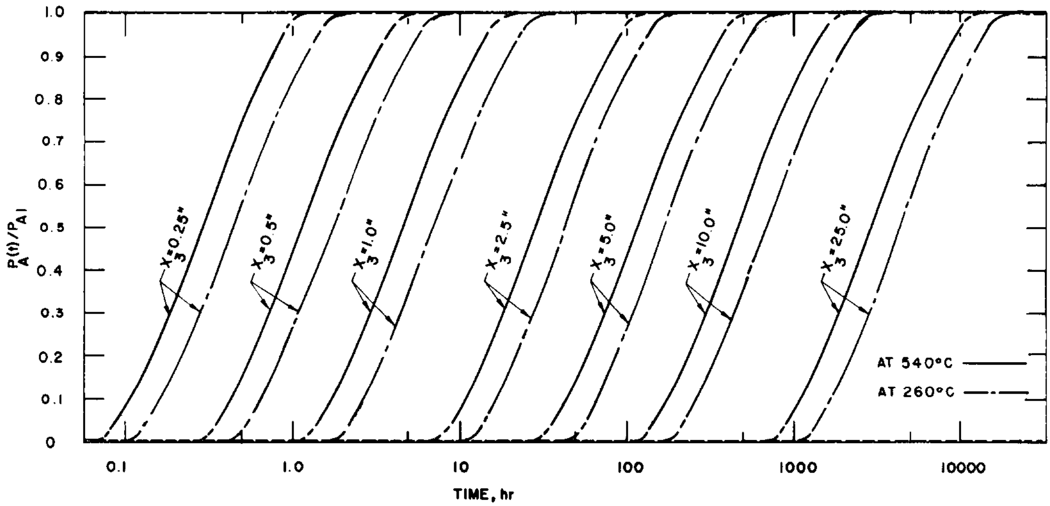

FIG. 2. Diffusion of argon gas into surface cavities at 260 and 540°C.

By employing equations (7), (10) and (11), the partial pressure in the surface cavity can be expressed,

$$P_A(t) = \frac{C_A(t)}{C_{A1}} P_{A1}. \tag{29}$$

Solutions have been obtained for a one-dimensional, constant temperature, sodium–argon system. It was also assumed that the partial pressure of inert-cover gas is increased in a step-wise fashion. The concentration of inert gas in the sodium at the argon–sodium interface increases an amount C_{A1} as the step change of partial pressure of inert gas is added to the system. The increase in partial pressure of inert gas in the cavity has been found as a function of the time after the step change and of the depth of the sodium, X_3. These results are illustrated in Fig. 2 for constant temperature, argon–sodium systems at 260°C and 540°C.

These results illustrate that the diffusion of the argon-cover gas into the cavities on the heated surface is significant. Hence, it is important for experimenters to conduct their

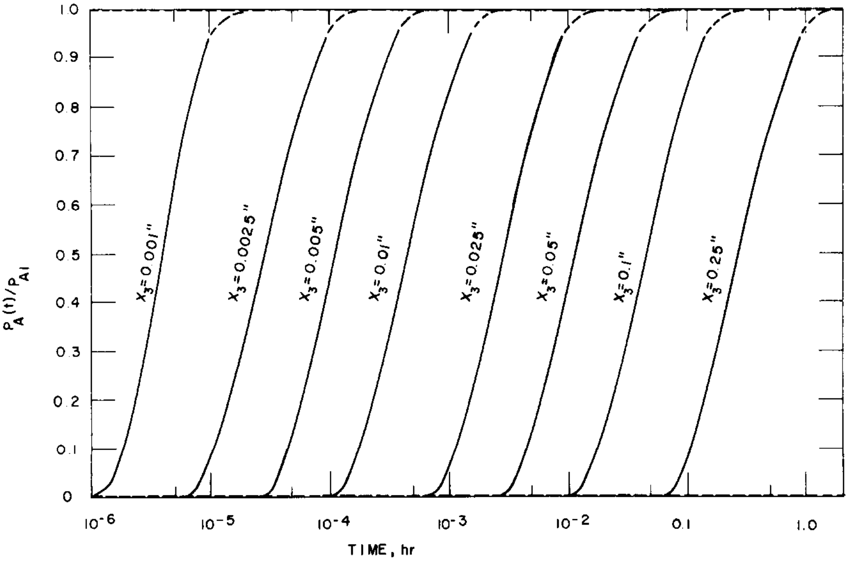

FIG. 3. Diffusion of argon gas into surface cavities in a flowing system at 540°C.

experimental sodium superheat tests in a manner so that knowledge of the inert-gas partial pressure in the cavities on the heater surface is available.

APPLICATION TO FLOWING SYSTEMS

In reactor systems and in experimental loops, the transport of inert gas is aided by convection. When forced convection effects are present (as would be expected in reactor systems or boiling sodium loops) the effects of inert-gas diffusion should be small compared to the transport of inert gas due to the liquid motion inherent in forced convection systems.

The buildup of inert-gas partial pressure in surface cavities (following a boiling run) can be calculated. A stationary molecular diffusion layer of thickness, X_3, between the vapor–gas region of a surface cavity and the flowing liquid is assumed. Also, sodium of the concentration, $C_{A1}(Y)$, in the mainstream is assumed, and as gas diffuses out of the flowing liquid, the liquid is instantaneously replaced with fresh liquid of the concentration, $C_{A1}(Y)$. Hence, only diffusion of inert gas through the molecular diffusion layer and into the unfilled portion of the surface cavities need be considered. Figure 3 illustrates the increase in inert-gas partial pressure in the surface cavities as a function of time.

Employing Henry's Law both at the free surface and at the local surface cavities and assuming steady-state conditions [i.e. $X_{gb} = X_A(Y)$], the inert-gas partial pressure in the surface cavities can be expressed,

$$P_A(Y) = \frac{K(T_{gb})\,P_{gb}}{K[T(Y)]}. \tag{30}$$

Hence, the partial pressure of inert gas in surface cavities is dependent upon both the inert-gas pressure and temperature at the free surface and the local temperature.

CONCLUSIONS

The results illustrate that the transport of the inert-cover gas into the cavities on the heater surface is a significant effect. Hence, it is important for experimenters to conduct their experimental sodium superheat tests in a manner so that knowledge of the inert gas partial pressure in the cavities on the heater surface is available.

Calculated results of inert-gas diffusion in a sodium–argon system illustrate that rather long times may be required for the inert-cover gas to diffuse into the surface cavities of an isothermal system. However, if forced convection effects are present (as would be expected in reactor systems or sodium-boiling experiments), the effects of inert-gas diffusion will be small when compared to the transport of the inert gas by convection. In this case, the actual times for inert gas to reach the surface cavity will be much shorter than those times calculated by assuming diffusion of the cover gas in an isothermal system.

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